Lecture 14. ***Research of absolute stability by the second (direct) method of Lyapunov***

Between Pópov's criterion of absolute stability and Lyapunov's second method there exists a profound connection. In 1962 *V.A. Yakubovich (В.А. Якубович)* proved that if Pópov's requirements of absolute stability are fulfilled, then in many case there exists Lyapunov's function which is equal to "square-law form plus integral of nonlinearity" as the following:

$$V(x) = X^T PX + b\int_0^a f(\xi)d\xi. \tag{6.22}$$

And requirement (6.21) is necessary and sufficient.

Let us consider a nonlinear system of indirect control. Let the system be given by the following equation:

$$\begin{cases} \dot{x} = Ax + bu & (6.23) \\ u = f(\sigma) & (6.24) \\ \dot{\sigma} = m^T x - \rho u & (6.25) \end{cases}$$

$$y = c^T x.$$

The following requirements are fulfilled:

$$f(0) = 0,$$

$$0 \le \frac{f(\sigma)}{\sigma} \le k.$$

It means that the working area includes the beginning of coordinates; and nonlinearity $f(\sigma)$ lies in angle *(0, k)* and passes through the beginning of coordinates (fig. 6.1). Nonlinearity is caused by introduction one nonlinear element into the linear system.

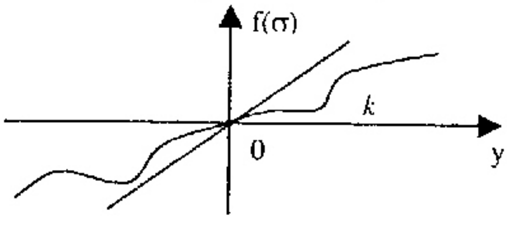


Fig. 6.1. Nonlinearity

It is accepted to classify nonlinear control problems into:

- control problems of direct type;

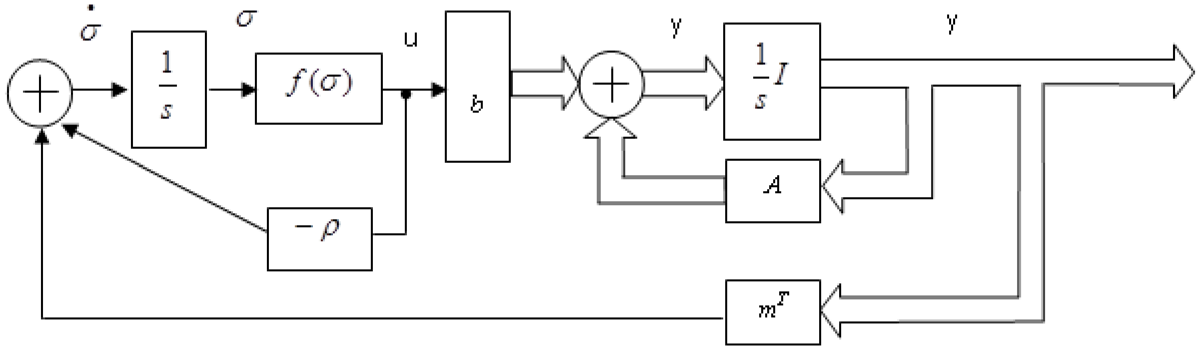
– indirect control problems.

In this lecture an indirect control system is investigated.

Let us realize transformation from coordinates (x, u) to coordinates (y, σ):

$$v = c^T x, \dot{y} = \dot{x},$$

$$\begin{cases} \dot{x} = Ax + bu \\ \dot{\sigma} = m^T x - \rho u \end{cases} \Rightarrow \begin{cases} y' = A_1 x + bf(\sigma) & (6.26) \\ \dot{\sigma} = m_y^T - \rho f(\sigma) & (6.27) \end{cases}$$



Pic. 6.2. Matrix scheme of indirect control system

Lyapunov's function for systems (6.26), (6.27) is chosen as the following:

$$V(y, \sigma) = y^T P y + \int_0^{\sigma} f(\sigma) d\sigma,$$

where $P = P^T$, $P > 0$ is a symmetrical matrix of fixed positive sign. The function must address to zero only at zero and be positive at other values.

Lyapunov's function consists of two items and satisfies to conditions of fixed positive sign according to (6.26), (6.27). Let us calculate the full differential of Lyapunov's function.

$$\frac{dV}{dt} = -y^T Q y + 2 f(\sigma) d^T y - \rho f^2(\sigma),$$

where *vector* $d = \rho b + \dfrac{m}{2}.$

THEOREM. If real parts of roots of the characteristic matrix A polynomial are negative

$$\operatorname{Re} S_i(A) < 0, \forall i = \overline{1, n},$$

zero solution of system (6.23), (6.24), (6.25) is absolute stable at the requirement

$$\rho > d^T Q^{-1} d,$$

where $Q = Q^T > 0$.

In fig. 6.2 there are presented matrix scheme of a nonlinear system of direct control.

Thus, both Pópov's theorem and Lyapunov's function of form (6.22) enclose the criteria of absolute stability as a set of nonlinear systems with one (or reducing to one) inertialless nonlinearity, characteristic of which corresponds to class *(0,k)*.